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Lacunarity analysis of fracture networks: Evidence for scale-dependent clustering

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ABSTRACT

Previous studies on fracture networks have shown that fractures contained within distinct mechanical units ("stratabound") are regularly spaced while those that terminate within the rock mass are clustered ("non-stratabound"). Lacunarity is a parameter which can quantify the distribution of spaces between rock fractures. When normalized to account for differences in fracture abundance, lacunarity characterizes the distribution of spaces as the degree of clustering in the fracture network. Normalized lacunarity curves, $L^*(r)$, computed using the gliding-box algorithm and plotted as a function of box-size, r, were constructed for natural fracture patterns from Telpyn Point, Wales and the Hornelen basin, Norway. The results from analysis of the Telpyn Point fractures indicate that such curves are sensitive to differences in the clustering of different fracture sets at the same scale. For fracture networks mapped at different scales from the Hornelen basin, our analysis shows that clustering increases with decreasing spatial scale. This trend is attributed to the transition from a "stratabound" system at the scale of sedimentary cycles (100–200 m) that act as distinct mechanical units to a "non-stratabound" fracture system geometry at the finer 10's of meters thick bedding scale.

1. Introduction

Fractures control or influence important behaviors in geological systems such as fluid storage, contaminant transport, seismicity, and rock strength. In the context of joints, a key attribute that influences these characteristics is the geometry of the fracture network. To better understand joint geometry it is necessary to consider fractures from the perspective of mechanical stratigraphy. Joints in sedimentary rocks fall in two categories, those that terminate randomly within the rock mass and those that terminate at distinct mechanical layer boundaries (Gross et al., 1995). Lithologic contacts, as well as pre-existing fractures, can serve as mechanical layer boundaries, thereby dividing the rock mass into discreet mechanical units (Gross, 1993). For our study, only lithologic contacts are considered as mechanical layer boundaries. Fractures that terminate at lithologic contacts are termed as "stratabound" while the ones that randomly terminate within the rock mass are "non-stratabound" (Odling et al., 1999; Gillespie et al., 1999). The former often display a log-normal distribution for length (Narr and Suppe, 1991) or other non-power law type distributions and appear to be regularly spaced as seen in the siliceous layers of the Monterey Formation (Gross et al., 1995). The "non-stratabound" fractures, however, have a wide range of length distributions (e.g. joint patterns at the Oliana anticline, Shackleton et al., 2005), sometimes yielding a power law, and are typically clustered (Odling et al., 1999; Gillespie et al., 1999).

Interface strength and the contrast between the rheology of layers control the ability of joints to propagate through lithologic contacts. Analog and numerical experiments suggest that weak interfaces inhibit joint propagation by sliding or opening, and similarly cracks terminate at contacts with soft and ductile layers (Shackleton et al., 2005 and references therein). In this case, the joints developed are "stratabound" and their spacing is proportional to the bed thickness (Narr and Suppe, 1991; Wu and Pollard, 1995 and references therein; Gross et al., 1995; Gillespie et al., 1999; Odling et al., 1999; Cooke et al., 2006). The driving condition for such joint formation is the result of either remote extension or possibly thermal relaxation (Hobbs, 1967; Engelder and Fischer, 1996; Bai and Pollard, 2000). In contrast, for stratabound joints the driving condition for fracture formation relates to fluid pressure (Gillespie et al., 1999; Odling et al., 1999; Engelder and Fischer, 1996).

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Joint spacing distributions can be measured from 1D scanlines (La Pointe and Hudson, 1985). Semi-variograms constructed from such measurements have been independently employed by La Pointe and Hudson (1985) and Chiles (1988) for quantifying the spatial heterogeneity of fracture networks. The ratio of the standard deviation to the mean of the spaces along a scanline has also been used by Gillespie et al. (1999) to discern between clustered and anticlustered veins. Given that rock properties can vary with direction, if possible it is more useful, although certainly more time consuming, to characterize joint spacing distribution in two dimensions using an area or map approach (Wu and Pollard, 1995; Rohrbaugh et al., 2002). In this paper, we present a technique modified from Plotnick et al. (1996) for analyzing clustering of joint populations in a two-dimensional representation.

To quantify the clustering of fractures, we use the concept of lacunarity (Mandelbrot, 1983). This approach is based on a multiscale analysis of spatial or temporal dispersion (Plotnick et al., 1996). Stated simply, lacunarity characterizes the distribution of spaces or gaps in a pattern as a function of scale. For a fracture pattern, therefore, it can be employed to quantify the degree of fracture clustering at a given spatial resolution. To implement lacunarity as a tool for our purpose, we have introduced a new normalization of this parameter. It is distinct from that of Plotnick et al. (1996) and completely removes the effect of fracture abundance on the lacunarity values. We use a set of three maps from Wales, U.K (Rohrbaugh et al., 2002) to demonstrate the usefulness of our normalized lacunarity measure over that proposed by Plotnick et al. (1996) and show its effectiveness in discerning between different sets of fractures within the same network. We then use normalized lacunarity to analyze a set of four maps from the Devonian sandstones of Hornelen basin, Norway (Odling, 1997) to investigate clustering of fractures at different scales. Finally, we interpret our observations from this sedimentary package in terms of mechanical stratigraphy as a function of scale.

2. Lacunarity and its quantification

A useful conceptual perspective for understanding lacunarity is to evoke the idea of translational invariance. Consider a uniform sequence of alternating 0's and 1's like 101010101… and so on. This sequence will map onto itself if a copy is made and moved over by two digits so that the original cannot be distinguished from the translated copy. This property is called translational invariance. In terms of lacunarity, a translationally invariant pattern exhibits no clustering, because all of the gap sizes (denoted by zeroes in our example) are the same. This behavior is not observed in the case of a slightly more heterogeneous sequence, such as 101000101… where the gaps have a range of sizes, including a cluster of three gaps in the middle. The greater the degree of gap clustering, the greater the lacunarity. Lacunarity is a scale-dependent parameter because sets that are uniform at a coarse scale might be heterogeneous at a finer scale, and vice-versa. Lacunarity can thus be considered as a scale-dependent measure of textural heterogeneity (Allain and Cloitre, 1991; Plotnick et al., 1993).

Quantifying lacunarity as a function of scale can be achieved by using the gliding-box algorithm (Allain and Cloitre, 1991; Plotnick et al., 1996). This algorithm slides a window or box of a given length, r, translated in increments of a chosen unit length across the pattern. In the case of all our analyses, this unit length is chosen to be at the pixel scale (size of the smallest dot that can be drawn on a computer screen). The box-size, r, is generally a multiple of this assigned unit length. The interrogator box searches for occupied sites in the pattern at each step and counts them as $s(r)$. The total number of steps, $N(r)$, required to cover the entire pattern is given by:

$$N(r) = (r_t - r + 1)^E \tag{1}$$

Here, E is the Euclidean dimension of the pattern (for fracture maps, $E = 2$) and r_t is the total length of the set. The first and second moments of the distribution of the number of occupied sites at each step, $Z_1(r)$, and $Z_2(r)$ respectively, are given by (Plotnick et al., 1996):

$$Z_1(r) = s(r) \tag{2a}$$

$$Z_2(r) = s_s^2(r) + [s(r)]^2 \tag{2b}$$

Here $s(r)$ and $s_s^2(r)$ are the arithmetic mean and variance of $s(r)$, respectively. The lacunarity is then defined as a function of box-size, $L(r)$, by (Allain and Cloitre, 1991):

$$L(r) \equiv Z_2(r)/[Z_1(r)]^2 \tag{3}$$

In terms of the mean and variance of $s(r)$ the lacunarity can also be expressed as:

$$L(r) = s_s^2(r)/[s(r)]^2 + 1 \tag{4}$$

Lacunarity is thus the dimensionless ratio of the dispersion (variance) to the square of the central tendency (mean) at a given scale, r (Plotnick et al., 1996). An alternative derivation of lacunarity may be found in Turcotte (1997).

Typically, lacunarity, $L(r)$, is calculated for a range of box-sizes r, and is plotted as a "lacunarity curve." For any given pattern, this curve will have upper and lower bounding values. Let ϕ be the fraction of sites that are occupied. It may then be proved that for $r = 1$, $Z_1(1) = \phi$ and $Z_2(1) = \phi$ in all cases (Plotnick et al., 1996). As a result, the lacunarity $L(1) = Z_2(1)/[Z_1(1)]^2 = \phi/\phi^2 = 1/\phi$. For $r = r_t$, there is only one box that covers the entire pattern and hence there the distribution of occupied sites, $s(r_t)$ consists of just one value. This implies that the variance, $s_s^2(r_t) = 0$. The lacunarity therefore is $L(r_t) = 1$. To summarize, the upper and lower bounds of the lacunarity curve are $L_{max} = L(1) = 1/\phi$ and $L_{min} = L(r_t) = 1$, respectively. The upper bound indicates that differences in ϕ will result in different values of L_{max}, and thus different lacunarity curves, even in the case of fracture patterns with similar clustering characteristics. The lacunarity parameter therefore needs to be normalized in order to overcome this effect.

3. Normalization of lacunarity: the Telpyn point fractures

The fracture network at Telpyn Point, UK, (Rohrbaugh et al., 2002) is comprised primarily of two orthogonal sets of vein-filled joints (striking 200° (NS trending) and 290° (EW trending)) that occur in Carboniferous sandstone (Dunne and North, 1990; Rohrbaugh et al., 2002) (Fig. 1a). The pattern was sampled over an area of 247.6 m^2. The NS-trending joints occur mainly in clusters (Fig. 1b), while the EW trending set consists of somewhat clustered, large joints (Fig. 1c).

The original fracture map from Rohrbaugh et al. (2002) was converted into three different maps (Fig. 1) each being a 545 × 578 pixel bitmap. The gliding-box technique, as outlined in Section 2, was applied to each map using a Matlab program (Roy, 2006) to generate the lacunarity curves (Fig. 2a). As seen in Fig. 2a, the EW fracture set yields much greater lacunarity values as compared to the NS set. This result is quite contrary to what is expected because visual inspection of the NS fractures (Fig. 1a) clearly indicates that they are more clustered than the EW set (Fig. 1b). This apparent discrepancy arises because the lacunarity values are controlled both by clustering and by the ϕ value, which correlates to the fracture abundance. Thus, patterns with a small

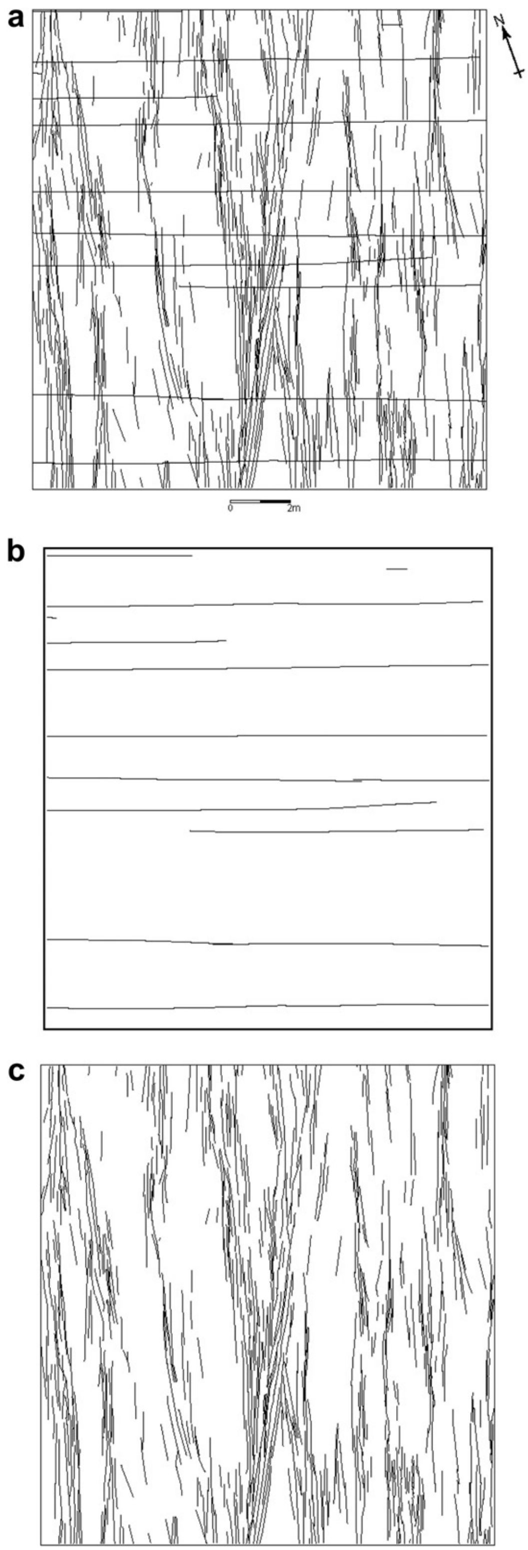

Fig. 1. Telpyn Point, Wales, fracture maps (Rohrbaugh et al., 2002): (a) NS trending fractures (b) EW trending fractures (c) both EW and NS trending fracture sets.

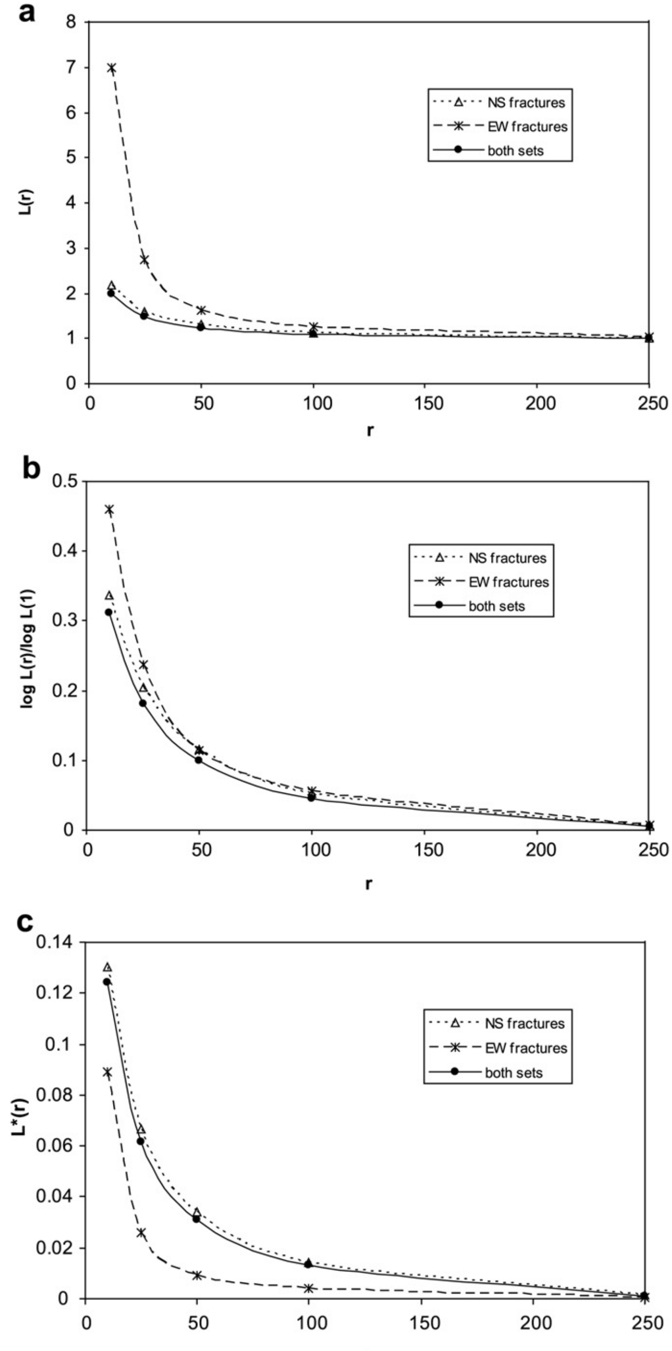

Fig. 2. (a) Non-normalized lacunarity curves for Telpyn Point fractures (b) same set of curves using Plotnick et al. (1996) normalization of lacunarity (c) same set of curves using new normalization, L^*.

fracture abundance (i.e. low ϕ-value) and therefore a high L_{max}, will tend to have a greater lacunarity as the size of the gliding box (r) goes to smaller values close to the size of a pixel. Clearly, the NS fractures (Fig. 1c) are more abundant than the EW fractures (Fig. 1b). Since the EW pattern has a ϕ-value that is seven times smaller than the NS pattern, the effect of the ϕ-value overrides the effect of clustering in the calculation of the lacunarity values.

In an attempt to eliminate the abundance effect, Plotnick et al. (1996) used the quotient of the log-transformed values of $L(r)$ and L_{max} to normalize the lacunarity function. We implemented their normalization approach for the Telpyn Point fracture maps and the results are plotted as $\log[L(r)]/\log[L_{max}]$ versus r in Fig. 2b. It can be seen that while this approach reduces the overall discrepancy, it does not eliminate it altogether; the EW fracture set still has the higher curve, again suggesting greater clustering. Therefore, we propose an alternative approach, widely used in the physical sciences, for normalizing the lacunarity parameter as:

$$L^*(r) = \frac{L(r) - L_{\min}}{L_{\max} - L_{\min}} = \frac{L(r) - 1}{1/\phi - 1} \tag{5}$$

where $L^*(r)$ is the normalized lacunarity. This normalization has two advantages. Firstly, the lacunarity does not need to be log-transformed because its values now range between unity at $r = 1$ to zero at $r = r_t$. Secondly, it completely removes the effect of the ϕ-value since the normalized lacunarity values reflect the effects of clustering alone rather than both clustering and fracture abundance. Compared to the curves for lacunarity in Fig. 2a and b, the curves for normalized lacunarity in Fig. 2c do show that the more clustered NS set has much higher normalized lacunarity values as compared to the sparsely spaced EW set.

Fig. 2 also includes lacunarity results for the NS and EW sets combined into a single network. Regardless of the technique employed, it is obvious that the lacunarity of the combined set is always dominated by the contribution of the NS set. In the case of our newly proposed normalization, the $L^*(r)$ curve for both sets combined is only slightly less than that for the single NS set. This result is because the NS fractures are very tightly clustered and, when combined with the sparsely spaced EW set, the character of the entire pattern is essentially controlled by the NS set.

4. Scale-dependent clustering: Hornelen basin fractures, Norway

4.1. Normalized lacunarity results

The Hornelen Basin fractures of Odling (1997) were chosen to delineate clustering within a fracture network at different scales. The four maps (Fig. 3) from this data set share two characteristics. They are all based on imagery gathered with a helicopter and they are a nested set of data where the sampling resolution changed with the change in map scale by varying the height of the helicopter. This approach is quite unlike collecting all data at one scale and then segmenting them to create maps at different scales. As a result, this pattern can be considered at a variety of scales in terms of the resolution of data at each scale, which is not the usual situation for the analysis of natural fracture patterns. The maps cover areas of sizes 90 m × 90 m (Map 4), 180 m × 180 m (Map 5), 360 m × 360 m (Map 6) and 720 m × 720 m (Map 7). Each map is a window on the fracture system and contains a range of fracture lengths, the shortest being dictated by the resolution of the image and the longest by the area mapped. When analyzed as fractal networks (Roy et al., 2007), the box-counting fractal dimensions, D_b, for each map were not statistically different (Table 1).

Table 1
Areas, scales, box-counting fractal dimensions, D_b, from Roy et al. (2007), fraction of sites occupied by fractures (ϕ), and non-normalized lacunarities $L(10)$ and $L(500)$ for Odling's (1997) fracture maps.

Map no.	Area (m^2)	Scale	D_b	ϕ	$L(10)$	$L(500)$
4	8100	1:511	1.81 ± 0.05	7.95	2.023	1.021
5	32,400	1:1023	1.82 ± 0.04	7.93	1.936	1.015
6	129,600	1:2045	1.84 ± 0.04	10.09	1.641	1.006
7	518,400	1:4091	1.84 ± 0.04	9.84	1.608	1.004

For lacunarity analysis, the original fracture maps of Odling (1997) were converted to 1042 × 1042 pixel bitmaps. Normalized lacunarity curves were computed for each of the four maps for five different r-values (Fig. 4). The lacunarity values for box-sizes of 10 and 500 pixels along with the ϕ-values of each map are documented in Table 1. Paired (two-tailed) t-tests performed between the $L^*(r)$ values of maps 4 and 5, 5 and 6, and 6 and 7 respectively, indicated that, when considered over all scales, the normalized lacunarities were significantly different at the 95% confidence level in each case. The trend revealed is: the greater the resolution (the smaller the map scale), the greater the lacunarity. This result implies that fractures are more clustered at small scales and more uniformly distributed at large scales.

4.2. Geologic interpretation

To geologically interpret the above results we need to return to a consideration of the differences between stratabound and non-stratabound joint networks. The former are ones that terminate at lithologic contacts while the latter terminate randomly within the rock mass and their geometries are not controlled by mechanical layer boundaries. Odling et al. (1999) cite the Hornelen fracture system as a good example of a "non-stratabound" fracture system, displaying joints with a power law length distribution and qualitatively observed clustered fractures with a lack of regular spacing. Our $L^*(r)$ curves quantitatively show that the Hornelen fracture system has evidence of decreasing clustering with increasing scale (Fig. 4). Each map represents a subset of the fracture system with respect to fracture length, implying that the fractures become less clustered with respect to each other as their length increases. This relationship suggests that the Hornelen fracture system tends towards a more "stratabound" type system as fractures approach the scale of the entire basin. Visual inspection of an aerial photograph of the Hornelen Basin, with long fractures (400–1500 m) and

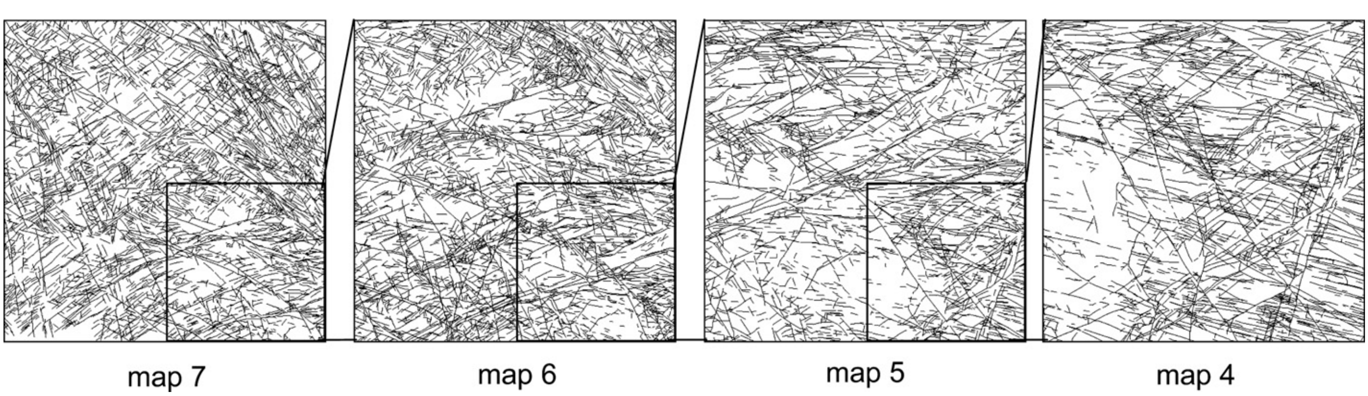

Fig. 3. Hornelen basin fracture network mapped from a helicopter (Odling, 1997): map 7 (720 m × 720 m), map 6 (360 m × 360 m), map 5 (180 m × 180 m) and map 4 (90 m × 90 m).

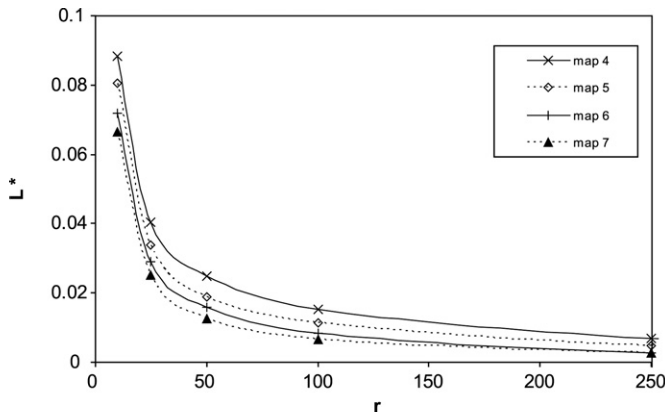

Fig. 4. Normalized lacunarity curves for Hornelen basin fracture maps 4,5,6 and 7 depicting scale-dependent clustering.

regular spacing (50–100 m), supports the tendency towards less clustering at larger scales (Fig. 5).

As discussed earlier, interface strength and the contrast between rheology of layers control the ability of joints to propagate through lithologic contacts. So, if the fracture system of the Hornelen Basin tends towards a smaller lacunarity (i.e. more "stratabound" system type), as scale increases and resolution decreases, the question of the nature of the layering that would control the fracture system at this greater scale arises. For map sizes of 90 m × 90 m to 720 m × 720 m, Odling (1997), Odling et al. (1999) and our results show that the fracture system is clustered. This condition implies that at these scales, the fractures are likely not stratabound and during formation likely propagated across bedding surfaces that had cohesion and lacked sufficient differences in mechanical properties between beds. Therefore, at the scale of bedding (10's of meters or less), the layers do not constitute distinct mechanical units which results in "non-stratabound" systems and noticeable lacunarity.

However, at the scale of sedimentary cycles, the lithological packages of the Hornelen Basin do have characteristic changes at the scale of 100–200 m of sequence. These packages are characterized by finer-grained material at their base (Steel, 1976), which results in a high rigidity contrast between the cycles. Therefore, as opposed to the bedding-scale layering, these cycles can be considered as distinct mechanical units that can house "stratabound" fractures. The cycles exert a strong control on the topography of the area which is clearly seen in the aerial photograph image (Fig. 5). From the aerial photograph, it seems that composite fractures large enough to penetrate the thickness of an individual cycle (lengths of 400–1500 m), tend to develop a more "stratabound" fracture system geometry with regular spacings of 50–100 m. The natural fracture patterns analyzed here (maps 4–7) were mapped from the well exposed surface of one of these cycles. The smallest map of 90 m × 90 m (map 4) shows a fracture length mode of around 1.7 m and a range of fracture lengths from 0.15 to 52 m. The majority of fractures in this map therefore have lengths comparable with the thickness of individual beds. Because the beds do not act as distinct mechanical units, this map shows a greater degree of clustering with a corresponding large lacunarity value. In the 720 × 720 m map (map 7), the fracture length mode is 11.7 m with a length range from 1.4 to 281 m. Thus, only the very largest fractures imaged by this map will penetrate an entire cycle which, as opposed to a single bed (10's of meters), act as a distinct mechanical unit. The progressive decrease in lacunarity as the scale increases (from map 4 to map 7), may therefore reflect an increasing influence of cycle thickness on the fracture system geometry. As the fracture lengths in the observed subset of the fracture system increase, the influence of cycle thickness (distinct mechanical unit) on fracture system geometry increases and the fracture system evolves from a "non-stratabound" type towards a more "stratabound" system. This change corresponds to a progressive reduction in lacunarity reflecting the transition from a clustered ("non-stratabound") to a more regularly spaced ("stratabound") fracture system.

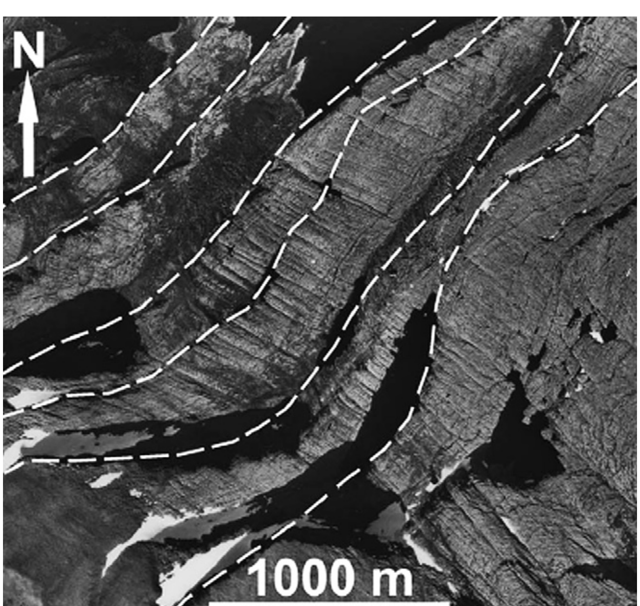

Fig. 5. Section of an aerial photograph from Hornelen showing typical regularly spaced fractures with lengths of 400–1500 m.

5. Conclusions

Plotnick et al. (1996) have shown that lacunarity is an effective means of characterizing spatial dispersion. Our present study shows that lacunarity can be used to quantify clustering in two-dimensional fracture networks. Procedurally, it refines the normalization technique of Plotnick et al. (1996) to account for differences in the fraction of occupied sites in fracture maps with varying fracture abundance.

Separate analyses of two different sets of fractures within the same network (Telpyn Point), as well as that for the combined sets, show that normalized lacunarity is more sensitive to clustering than either the non-normalized lacunarity or Plotnick et al. (1996) previous normalization. We also demonstrated that the normalized lacunarity can quantify the degree of clustering so as to reveal that the most tightly clustered set controls the lacunarity curve of the pattern as a whole.

The normalized lacunarity for the complex, multi-generational pattern of Hornelen basin fractures clearly indicates that fractures become more clustered (like "non-stratabound" type) as the spatial scale of observation is decreased. Additional observations at the aerial-photograph scale show that fractures, which possibly penetrate the entire thickness of major sedimentary cycles (100–200 m), are regularly spaced at 50–100 m like "stratabound" fractures. This observation implies that these cycles behave like distinct mechanical units as opposed to the beds (10's of meters thick) that are contained within them. It is argued that this trend reflects a gradual evolution from a "non-stratabound" fracture network, with greater clustering at the bed scale, towards a more "stratabound" system, with lesser clustering as fracture size perpendicular to bedding approaches the thickness of major sedimentary cycles.

Since fracture patterns can generally only be examined over a limited range of scales, such as with seismic reflection data, our results could be economically important for the mining and petroleum industries. Any scale-dependency in the clustering of fractures will also likely have significant implications for rock strength and flow processes that depend upon fracture connectivity. Thus, in terms of potential consequences, the nature of the relationship between lacunarity and fracture connectivity deserves to be elucidated in future studies.

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